

A Generalized S-Transform in Linear Canonical Transform

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Submission date: 18-Jan-2023 11:29AM (UTC+0700)

Submission ID: 1994572673

File name: Bahri_2019_J._Phys._Conf._Ser._1341_062005.pdf (524.66K)

Word count: 2774

Character count: 12037

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To cite this article: Mawardi Bahri *et al* 2019 *J. Phys.: Conf. Ser.* **1341** 062005

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A Generalized S-Transform in Linear Canonical Transform

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Abstract. The linear canonical S-transform is a generalization of the classical S-transform using linear canonical transform. In the present work, we propose the definition of the linear canonical S-transform and investigate important its properties.

1. Introduction

Recently many works have concentrated on constructing various types of transformations using the linear canonical transform. In [1, 2], the authors have proposed the windowed linear canonical transform which is an extension of the classical windowed Fourier transform using the linear canonical transform. Some basic properties of generalized transform were investigated in detail. Papers [3, 4, 5, 6, 7, 8, 9] have studied the Wigner-Ville distribution associated with the linear canonical transform and the offset linear canonical transform. Several important properties of these extended transformations were also demonstrated. In [10], the authors shortly introduced S-transform associated with the discrete version of the linear canonical transform and its application to signal analysis. Our purpose of this work is to extend the S-transform to the linear canonical transform, which we shall call the linear canonical S-transform (LCST). We build its important properties, which are modifications of the properties of the traditional S-transform.

Definition 1.1. ([11, 12, 13, 14]) Given any function $h \in L^1(\mathbb{R})$. Let $N = \begin{bmatrix} m & n \\ p & q \end{bmatrix} \in \mathbb{R}^{2 \times 2}$ be a matrix parameter such that $\det(N) = mq - np = 1$. The linear canonical transform of f is defined by

$$L_N\{h\}(\omega) = \begin{cases} \int_{-\infty}^{\infty} h(x)K_N(\omega, x) dx, & n \neq 0 \\ \sqrt{|q|}e^{i\frac{pq}{2}\omega^2}h(q\omega), & n = 0, \end{cases} \quad (1)$$

where $K_N(x, \omega)$ is so-called kernel of the LCT given by

$$K_N(x, \omega) = \frac{1}{\sqrt{2\pi n}} e^{i\frac{1}{2}\left(\frac{m}{n}x^2 - \frac{2}{n}x\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2}\right)}.$$

¹

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Here it should be noticed that the LCT kernel mentioned above has the following important property

$$K_{N^{-1}}(\omega, x) = \overline{K_N(x, \omega)} = \frac{1}{\sqrt{2\pi n}} e^{-\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2})}.$$

The above definition gives us information that for $n = 0$ the LCT of a signal is essentially a chirp multiplication. Therefore, in this paper we always assume $n \neq 0$.

It is known that given $L_N\{h\}$ we can obtain h by the inverse formula of the LCT given by

$$\begin{aligned} L_N^{-1}[L_N\{h\}](x) &= h(x) = \int_{-\infty}^{\infty} L_N\{h\}(\omega) K_N^{-1}(\omega, x) d\omega \\ &= \int_{-\infty}^{\infty} L_N\{h\}(\omega) \frac{1}{\sqrt{2\pi n}} e^{-\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2})} d\omega. \end{aligned} \quad (2)$$

2. Definition of Linear Canonical S-Transform

Like the Fourier and wavelet transforms, the S-transform is a time-frequency localization technique, which was first proposed by Stockwell et al. [15]. In [16], the author investigated the concept of the S-transform in distribution space. In the following we extend the S-transform to the linear canonical transform called the linear canonical S-transform (LCST). We first investigate some consequences of the LCST definition.

Definition 2.1 (LCST Definition). *Let $\phi \in L^2(\mathbb{R})$ be a non-zero window function. Denote by \mathcal{S}_ϕ^N , the LCST on $L^2(\mathbb{R})$. The LCST of $h \in L^2(\mathbb{R})$ with respect to ϕ is defined as*

$$\begin{aligned} \mathcal{S}_\phi^N h(u, \omega) &= \int_{-\infty}^{\infty} h(x) \overline{\phi(u-x, \omega)} K_N(x, \omega) dx \\ &= \frac{1}{\sqrt{2\pi n}} \int_{-\infty}^{\infty} h(x) \overline{\phi(u-x, \omega)} e^{\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2})} dx. \end{aligned} \quad (3)$$

It is straightforward to check that when $N = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, the LCST reduces to S-transform given by

$$\mathcal{S}_\phi h(u, \omega) = \frac{e^{-i\frac{\pi}{4}}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} h(x) \overline{\phi(u-x, \omega)} e^{-i\omega x} dx. \quad (4)$$

We summarize the following consequences of the above definition.

- It is easy to see that

$$\mathcal{S}_\phi^N h(u, \omega) = L_N\{h(x) \overline{\phi(u-x, \omega)}\}(\omega, u), \quad (5)$$

- Using the inverse LCT to (3) we get

$$\begin{aligned} h(x) \overline{\phi(u-x, \omega)} &= L_N^{-1}\{\mathcal{S}_\phi^N h(u, \omega)\} \\ &= \int_{-\infty}^{\infty} \mathcal{S}_\phi^N h(u, \omega) \frac{1}{\sqrt{2\pi n}} e^{-\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2})} d\omega. \end{aligned} \quad (6)$$

- If $\phi \in L^1(\mathbb{R}) \cap L^2(\mathbb{R})$ satisfying

$$\int_{-\infty}^{\infty} \phi(u-x, \omega) du = 1. \quad (7)$$

Then for every $h \in L^2(\mathbb{R})$

$$\begin{aligned} \int_{-\infty}^{\infty} \mathcal{S}_{\phi}^N h(u, \omega) du &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x) \overline{\phi(u-x, \omega)} K_N(x, \omega) du dx \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x) \overline{\phi(u-x, \omega)} \frac{1}{\sqrt{2\pi n}} e^{\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2})} du dx \\ &= \int_{-\infty}^{\infty} h(x) \frac{1}{\sqrt{2\pi n}} e^{\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2})} \int_{-\infty}^{\infty} \overline{\phi(u-x, \omega)} du dx \\ &= L_N\{h\}(\omega). \end{aligned} \tag{8}$$

- If $\phi(u-x, \omega) = m(\omega)$, then

$$\begin{aligned} \mathcal{S}_{\phi}^N h(u, \omega) &= \int_{-\infty}^{\infty} h(x) \overline{\phi(u-x, \omega)} K_N(x, \omega) dx \\ &= m(\omega) \int_{-\infty}^{\infty} h(x) \frac{1}{\sqrt{2\pi n}} e^{\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2})} dx \\ &= m(\omega) L_N\{h\}(\omega). \end{aligned} \tag{9}$$

3. Basic Properties of Linear Canonical S-Transform

The following results describe several useful properties of the linear canonical S-transform, which have not been established in [10]. As in the case of the windowed linear canonical transform, we obtain that most properties of the classical S-transform can be established in the linear canonical S-transform domain with some modifications. However, it is found that the properties of the LSCF like shift and modulation differ from the corresponding properties of the windowed linear canonical transform (compare to [1, 2]).

Theorem 3.1. *Let $\phi \in L^2(\mathbb{R})$ be a complex window function. The linear canonical S-transform of $h, g \in L^2(\mathbb{R})$ is a linear operator, which means that*

$$\mathcal{S}_{\phi}^N(\alpha h + \beta g)(u, \omega) = \alpha \mathcal{S}_{\phi}^N h(u, \omega) + \beta \mathcal{S}_{\phi}^N g(u, \omega), \tag{10}$$

for arbitrary constants α and β .

Proof. It is straightforward to verify. □

Theorem 3.2. *If $\phi \in L^2(\mathbb{R})$ is a complex window function, then we have*

$$\mathcal{S}_{P\phi}^N\{Ph\}(u, \omega) = \mathcal{S}_{\phi}^N h(-u, -\omega), \tag{11}$$

where $P\phi(x) = \phi(-x)$.

Proof. A direct calculation gives for every $h \in L^2(\mathbb{R})$

$$\begin{aligned} \mathcal{S}_{P\phi}^N\{Ph\}(u, \omega) &= \int_{-\infty}^{\infty} h(-x) \overline{\phi(-(u-x), -\omega)} \frac{1}{\sqrt{2\pi n}} e^{\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2})} dx \\ &= \int_{-\infty}^{\infty} h(-x) \overline{\phi(-u - (-x), -\omega)} \frac{1}{\sqrt{2\pi n}} e^{\frac{i}{2}(\frac{m}{n}(-x)^2 - \frac{2}{n}(-x)(-\omega) + \frac{q}{n}(-\omega)^2 - \frac{\pi}{2})} dx, \end{aligned} \tag{12}$$

which the theorem follows. □

Theorem 3.3. Let $\phi \in L^2(\mathbb{R})$ be a complex window function. Then the following property is satisfied

$$\mathcal{S}_\phi^N(T_{x_0}h)(u, \omega) = e^{\frac{i}{2}\frac{m}{n}x_0^2} e^{-i\frac{ix_0\omega}{n}} \mathcal{S}_\phi^N \bar{h}(u - x_0, \omega), \quad (13)$$

where $\bar{h}(t) = e^{i\frac{mtx_0}{n}} h(t)$ and $T_{x_0}h(x) = h(x - x_0)$.

Proof. By taking into account of (3), we get

$$\mathcal{S}_\phi^N(T_{x_0}h)(u, \omega) = \int_{-\infty}^{\infty} h(x - x_0) \overline{\phi(u - x, \omega)} \frac{1}{\sqrt{2\pi n}} e^{\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2})} dx. \quad (14)$$

Using the change of variables $t = x - x_0$ yields

$$\begin{aligned} & \mathcal{S}_\phi^N(T_{x_0}h)(u, \omega) \\ &= \int_{-\infty}^{\infty} h(t) \overline{\phi(u - (t + x_0), \omega)} \frac{1}{\sqrt{2\pi n}} e^{\frac{i}{2}(\frac{m}{n}(t+x_0)^2 - \frac{2}{n}(t+x_0)\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2})} dt \\ &= \int_{-\infty}^{\infty} h(t) \overline{\phi((u - x_0) - t, \omega)} \frac{1}{\sqrt{2\pi n}} e^{i(\frac{1}{2}\frac{m}{n}t^2)} e^{i(\frac{1}{2}\frac{2m}{n}tx_0)} e^{i(-\frac{1}{2}\frac{2t\omega}{n})} e^{i(\frac{1}{2}\frac{m}{n}x_0^2)} e^{i(-\frac{1}{2}\frac{2x_0\omega}{n})} e^{i(\frac{1}{2}\frac{q}{n}\omega^2)} e^{-i\frac{\pi}{2}} dt \\ &= e^{\frac{i}{2}\frac{m}{n}x_0^2} e^{-i\frac{ix_0\omega}{n}} \int_{-\infty}^{\infty} e^{i\frac{mtx_0}{n}} h(t) \overline{\phi((u - x_0) - t, \omega)} \frac{1}{\sqrt{2\pi n}} e^{\frac{i}{2}(\frac{m}{n}t^2 - \frac{2}{n}t\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2})} dt. \end{aligned} \quad (15)$$

This completes the proof of theorem. \square

Theorem 3.4. For any function $h \in L^2(\mathbb{R})$ and complex window function $\phi \in L^2(\mathbb{R})$, we have

$$\mathcal{S}_\phi^N(\mathcal{M}_{\omega_0}h)(u, \omega) = e^{iq\omega\omega_0} e^{-i\frac{1}{2}q\omega\omega_0^2} \int_{-\infty}^{\infty} h(x) \overline{\phi((u - x), \omega)} K_N(\omega - \omega_0 n, x) dx, \quad (16)$$

where $\mathcal{M}_{\omega_0}h(x) = e^{i\omega_0 x} h(x)$.

Proof. In view of (3) a direct calculation shows that

$$\begin{aligned} & \mathcal{S}_\phi^N(\mathcal{M}_{\omega_0}h)(u, \omega) \\ &= \int_{-\infty}^{\infty} e^{i\omega_0 x} h(x) \overline{\phi((u - x), \omega)} \frac{1}{\sqrt{2\pi n}} e^{\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2})} dx \\ &= \int_{-\infty}^{\infty} h(x) \overline{\phi((u - x), \omega)} \frac{1}{\sqrt{2\pi n}} e^{\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x\omega + \frac{q}{n}\omega^2 + 2\omega_0 x - \frac{\pi}{2})} dx \\ &= \int_{-\infty}^{\infty} h(x) \overline{\phi((u - x), \omega)} \frac{1}{\sqrt{2\pi n}} e^{\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x(\omega - \omega_0 n) + \frac{q}{n}\omega^2 - \frac{\pi}{2})} dx \\ &= \int_{-\infty}^{\infty} h(x) \overline{\phi((u - x), \omega)} \frac{1}{\sqrt{2\pi n}} e^{\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x(\omega - \omega_0 n) + \frac{q}{n}((\omega - \omega_0 n) + \omega_0 n)^2 - \frac{\pi}{2})} dx \\ &= \int_{-\infty}^{\infty} h(x) \overline{\phi((u - x), \omega)} \frac{1}{\sqrt{2\pi n}} e^{\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x(\omega - \omega_0 n) + \frac{q}{n}((\omega - \omega_0 n)^2 - \frac{\pi}{2} + 2(\omega - \omega_0 n)\omega_0 n + \omega_0^2 n^2))} dx \\ &= e^{id\omega\omega_0} e^{-i\frac{1}{2}q\omega\omega_0^2} \int_{-\infty}^{\infty} h(x) \overline{\phi((u - x), \omega)} \frac{1}{\sqrt{2\pi n}} e^{\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x(\omega - \omega_0 n) + \frac{q}{n}(\omega - \omega_0 n)^2 - \frac{\pi}{2})} dx \\ &= e^{iq\omega\omega_0} e^{-i\frac{1}{2}q\omega\omega_0^2} \int_{-\infty}^{\infty} h(x) \overline{\phi((u - x), \omega)} K_N(\omega - \omega_0 n, x) dx, \end{aligned}$$

which completes the proof. \square

Theorem 3.5. Let $h \in L^2(\mathbb{R})$ be a complex function. If ϕ is a real window function, then

$$\mathcal{S}_\phi^N \bar{h}(u, \omega) = \overline{\mathcal{S}_\phi^{M-1} h(u, \omega)}. \quad (17)$$

Proof. It follows from (3) that

$$\begin{aligned} \mathcal{S}_\phi^N \bar{h}(u, \omega) &= \int_{-\infty}^{\infty} \overline{h(x)} \phi((u-x), \omega) \frac{1}{\sqrt{2\pi n}} e^{\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2})} dx \\ &= \int_{-\infty}^{\infty} h(x) \overline{\phi(u-x, \omega)} \frac{1}{\sqrt{2\pi n}} e^{-\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2})} dx \\ &= \overline{\mathcal{S}_\phi^{M-1} h(u, \omega)}. \end{aligned}$$

The proof is complete. \square

4. Fundamental Properties of Linear Canonical S-Transform

We are going to investigate fundamental properties of the LCST. We first develop the orthogonality relation associated with the LCST. Applying this property we then derive its reconstruction formula, which tells us that it is possible to restore the original signal h perfectly using the inverse LCST.

Theorem 4.1 (Orthogonality relation). Let ϕ be a complex window function. If $h, g \in L^2(\mathbb{R})$ are two complex functions arbitrary, then we have

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{S}_\phi^N h(u, \omega) \overline{\mathcal{S}_\psi^N g(u, \omega)} d\omega du = \left(h \int_{-\infty}^{\infty} |\phi(u, \omega)|^2 du, g \right). \quad (18)$$

In particular, for $h = g$ we obtain

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\mathcal{S}_\phi^N h(u, \omega)|^2 d\omega du = \|h\|_{L^2(\mathbb{R})}^2 \int_{-\infty}^{\infty} |\phi(u, \omega)|^2 du. \quad (19)$$

Proof. By the LCST definition (3), we have

$$\begin{aligned} (\mathcal{S}_\phi^N h, \mathcal{S}_\phi^N g) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{S}_\phi^N h(u, \omega) \overline{\mathcal{S}_\phi^N g(u, \omega)} du d\omega \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x) \overline{\phi(u-x, \omega)} \frac{1}{\sqrt{2\pi n}} e^{-\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2})} \\ &\quad \times \int_{-\infty}^{\infty} \overline{g(x)} \phi(u-t, \omega) \frac{1}{\sqrt{2\pi n}} e^{\frac{i}{2}(\frac{m}{n}t^2 - \frac{2}{n}t\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2})} dt dx du d\omega \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x) \overline{\phi(u-x, \omega)} \frac{1}{\sqrt{2\pi n}} e^{-\frac{mi}{2n}(x^2-t^2)} \\ &\quad \times \int_{-\infty}^{\infty} \overline{g(x)} \phi(u-t, \omega) \frac{1}{\sqrt{2\pi n}} e^{\frac{i}{n}(t-x)\omega} dt dx du d\omega \\ &= 2\pi n \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x) \overline{\phi(u-x, \omega)} \frac{1}{2\pi n} e^{-\frac{mi}{2n}(x^2-t^2)} \overline{g(x)} \phi(u-t, \omega) \delta(t-x) dt dx du \\ &= \int_{-\infty}^{\infty} h(t) \int_{-\infty}^{\infty} \overline{\phi(u-t, \omega)} \phi(u-t, \omega) \overline{g(t)} du dt \\ &= \int_{-\infty}^{\infty} h(t) \overline{g(t)} dt \int_{-\infty}^{\infty} \overline{\phi(u-t, \omega)} \phi(u-t, \omega) du \end{aligned}$$

$$\begin{aligned} &= \int_{-\infty}^{\infty} h(t)\overline{g(t)} dt \int_{-\infty}^{\infty} |\phi(u, \omega)|^2 du \\ &= \left(h \int_{-\infty}^{\infty} |\phi(u, \omega)|^2 du, g \right), \end{aligned} \quad (20)$$

which completes the proof. \square

In the next theorem, let us derive reconstruction formula associated with the linear canonical S-transform.

Theorem 4.2 (Reconstruction formula). *Let $\phi \in L^2(\mathbb{R})$ be a complex window functions such that ϕ satisfies the condition*

$$\int_{-\infty}^{\infty} |\phi(u, \omega)|^2 du = \mathcal{K}_\phi, \quad 0 < \mathcal{K}_\phi < \infty. \quad (21)$$

Then, every signal $h \in L^2(\mathbb{R})$ can be recovered using the so-called reconstruction formula, that is,

$$h(x) = \frac{1}{\mathcal{K}_\phi \sqrt{2\pi n}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{S}_\phi^N h(u, \omega) \phi(u - x, \omega) e^{-\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2})} d\omega du. \quad (22)$$

Proof. An application of (18) yields

$$\begin{aligned} &\left(h \int_{-\infty}^{\infty} |\phi(u, \omega)|^2 du, g \right) \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{S}_\phi^N h(u, \omega) \overline{\mathcal{S}_\phi^N g(u, \omega)} du d\omega \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{S}_\phi^N h(u, \omega) \int_{-\infty}^{\infty} \overline{g(x)} \phi(u - x, \omega) \frac{1}{\sqrt{2\pi n}} e^{-\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2})} dx du d\omega \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{S}_\phi^N h(u, \omega) \phi(u - x, \omega) \frac{1}{\sqrt{2\pi n}} e^{-\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2})} \overline{g(x)} du d\omega dx \\ &= \left(\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{S}_\phi^N h(u, \omega) \phi(u - x, \omega) \frac{1}{\sqrt{2\pi n}} e^{-\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2})} du d\omega, g \right). \end{aligned} \quad (23)$$

Because equation (23) is valid for every $g \in L^2(\mathbb{R})$ we get

$$h(x) \int_{-\infty}^{\infty} |\phi(u, \omega)|^2 du = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{S}_\phi^N h(u, \omega) \phi(u - x, \omega) \frac{1}{\sqrt{2\pi n}} e^{-\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2})} d\omega du. \quad (24)$$

Hence,

$$h(x) = \frac{1}{\mathcal{K}_\phi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{S}_\phi^N h(u, \omega) \phi(u - x, \omega) \frac{1}{\sqrt{2\pi n}} e^{-\frac{i}{2}(\frac{m}{n}x^2 - \frac{2}{n}x\omega + \frac{q}{n}\omega^2 - \frac{\pi}{2})} d\omega du. \quad (25)$$

Therefore, the proof is complete. \square

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